Characterizing False-name-proof Allocation Rules in Combinatorial Auctions

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ABSTRACT

A combinatorial auction mechanism consists of an allocation rule that defines the allocation of goods for each agent, and a payment rule that defines the payment of each winner. There have been several studies on characterizing strategy-proof allocation rules. In particular, a condition called *weak-monotonicity* has been identified as a full characterization of strategy-proof allocation rules. More specifically, for an allocation rule, there exists an appropriate payment rule so that the mechanism becomes strategy-proof if and only if it satisfies weak-monotonicity.

In this paper, we identify a condition called *sub-additivity* which characterizes false-name-proof allocation rules. False-name-proofness generalizes strategy-proofness, by assuming that a bidder can submit multiple bids under fictitious identifiers. As far as the authors are aware, this is the first attempt to characterize false-name-proof allocation rules. We can utilize this characterization for developing a new false-name-proof mechanism, since we can concentrate on designing an allocation rule. As long as the allocation rule satisfies weak-monotonicity and sub-additivity, there always exists an appropriate payment rule. Furthermore, by utilizing the sub-additivity condition, we can easily verify whether a mechanism is false-name-proof. To our surprise, we found that two mechanisms, which were believed to be false-name-proof, do not satisfy sub-additivity; they are not false-name-proof. As demonstrated in these examples, our characterization is quite useful for mechanism verification.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems*; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Theory, Economics

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Keywords

Mechanism Design, Combinatorial Auctions, False-name-proof

1. INTRODUCTION

Mechanism design of combinatorial auctions has become an integral part of Electronic Commerce and a promising field for applying AI and agent technologies. Among various studies related to Internet auctions, those on combinatorial auctions have lately attracted considerable attention (an extensive survey is presented in [4, 14]). Mechanism design is the study of designing a rule/protocol that achieves several desirable properties assuming that each agent hopes to maximize his own utility. One desirable property of an auction mechanism is that it is *strategy-proof.* A mechanism is strategy-proof if, for each bidder, declaring his true valuation is a *dominant strategy*, i.e., an optimal strategy regardless of the actions of other bidders. In theory, the revelation principle states that in the design of an auction mechanism, we can restrict our attention to strategy-proof mechanisms without loss of generality [12].

An auction mechanism consists of an allocation rule that defines the allocation of goods for each agent, and a payment rule that defines the payment of each winner. There have been many studies on characterizing strategy-proof social choice function (an allocation rule in auctions) in the literature of social choice theory [7]. This is also called the implementability of social choice functions. If a social choice function is implementable, we can find an appropriate payment rule so that the mechanism (the social choice function and the payment rule) becomes strategy-proof.

In particular, a family of *monotonicity* concepts was identified to characterize implementable social choice functions. For example, Rochet proposed a *cycle monotonicity* condition and showed that an allocation rule is strategy-proof if and only if this condition holds [15]. Bikhchandani *et al.* and Lavi *et al.* introduced a weaker notion of cycle monotonicity called *weak-monotonicity* and showed that it is necessary and sufficient for strategy-proof mechanisms under several assumptions on possible types [3, 5, 10, 16].

Such a characterization of allocation rules is quite useful for developing/verifying strategy-proof mechanisms. These conditions are defined only on an allocation rule; i.e., if it satisfies such a condition, it is guaranteed that there exists an appropriate payment rule that achieves strategy-proofness. Thus, a mechanism designer can concentrate on the allocation rule when developing/verifying mechanisms.

In fact, Babaioff and Blurmrosen developed a computationally feasible strategy-proof auctions for convex bundles.

By using monotonicity, they proposed a method to convert feasible algorithms into strategy-proof mechanisms [2, 1]. Parkes and Duong [13] used monotonicity in their ironing-based approach for online adaptive mechanisms. Lavi and Swamy [6] applied cycle monotonicity to develop strategy-proof mechanisms for scheduling.

On the other hand, Yokoo *et al.* pointed out the possibility of a new type of fraud called false-name bids (manipulations) that utilizes the anonymity available on the Internet [19, 20]. False-name bids are bids submitted under fictitious names, e.g., multiple e-mail addresses. Such dishonesty is very difficult to detect, since identifying each participant on the Internet is virtually impossible.

A mechanism is *false-name-proof* if, for each bidder, declaring his true valuations using a single identifier (although the bidder can use multiple identifiers) is a dominant strategy. False-name-proofness is a generalization of strategy-proofness. It is shown that the theoretically well-founded Vickrey-Clarke-Groves mechanism is not false-name-proof [20]. Furthermore, there exists no false-name-proof, Pareto efficient mechanism [20]. So far, several false-name-proof mechanisms have been developed [19, 17, 8]. However, there exists virtually no work on characterizing false-name-proof mechanisms. One notable exception is a characterization called *Price-Oriented, Rationing-Free (PORF)* mechanism presented in [17], that provides a condition on payments so that a mechanism is false-name-proof.

As far as the authors are aware, this paper is the first attempt to characterize false-name-proof allocation rules in combinatorial auctions. We first identify a condition called *sub-additivity* and then prove that we can find an appropriate payment rule if and only if the allocation rule simultaneously satisfies weak-monotonicity and sub-additivity.

This characterization can be utilized for several different purposes. First, we can utilize it for developing a new false-name-proof mechanism, since we can concentrate on designing an allocation rule, while so far central research questions tend to address how to design an appropriate payment rule (for a given allocation rule). As long as the designed allocation rule satisfies weak-monotonicity and sub-additivity, it is automatically guaranteed that there exists an appropriate payment rule to make the mechanism false-name-proof.

Second, we can utilize this characterization for clarifying the theoretical properties of false-name-proof mechanisms. For example, it is well-known that there exists no false-name-proof mechanism that always achieves a Pareto efficient allocation. However, we don't know the upper-bound of possible social surplus for false-name-proof mechanisms yet. By utilizing sub-additivity, we can concentrate on allocation rules to examine possible social surplus.

Finally, by utilizing the sub-additivity condition, we can easily verify whether a mechanism is false-name-proof, since the sub-additivity condition can be checked by fixing the types of other bidders and by changing the type of a single bidder (and his possible false-names). In short, we can check the sub-additivity condition by examining only the local behaviors of the allocation rule.

We have actually verified existing false-name-proof mechanisms and found that two mechanisms, GM-SMA[18] and Matsuo mechanism[8], which were believed to be false-name-proof, do not satisfy sub-additivity (thus they cannot be false-name-proof). As demonstrated in these examples, our characterization is quite useful for verifying false-name-proof mechanisms. Our characterization can detect a subtle failure of a mechanism, which is very difficult to do if we rely on standard proof techniques for false-name-proofness.

This paper is organized as follows. Section 2 describes our model and summarizes the existing results on the characterization of allocation rules. Section 3 introduces a condition called *sub-additivity* and proves that it is necessary and sufficient for false-name-proofness. Section 4 examines whether the sub-additivity condition holds in existing allocation rules. Section 5 concludes this paper.

2. PRELIMINARIES

Assume there exists a set of bidders $N = \{1, 2, \ldots, n\}$ and a set of goods $G = \{1, 2, \ldots, m\}$. Each bidder i has his preferences over $B \subseteq G$. Formally, we model this by supposing that bidder i privately observes a parameter, or signal, θ_i which determines his preferences. We refer to θ_i as the *type* of bidder i and assume it is drawn from a set Θ. We also assume a *quasi-linear, private value* model with *no allocative externality*, defined as follows:

Definition 1 (utility of a bidder).
The utility of bidder i, when i obtains a bundle, i.e., a subset of goods $B \subseteq G$ and pays p, is represented as $v(\theta_i, B) - p$.

We assume a valuation v is normalized by $v(\theta_i, \emptyset) = 0$. Furthermore, we assume *free disposal*, i.e., $v(\theta_i, B') \geq v(\theta_i, B)$ for all $B' \supseteq B$. Also, we assume Θ satisfies a condition called a *rich domain* [3], i.e., $\forall B \neq \emptyset, \forall c \geq 0, \exists \theta_i \in \Theta$, where $v(\theta_i, B) = c$ holds. In other words, the domain of types Θ is rich enough to contain all possible valuations. We require this assumption so that weak-monotonicity characterizes strategy-proofness.

A combinatorial auction mechanism $\mathcal{M}$ consists of an allocation rule $X : \Theta^n \to P_G^n$, where P_G is a power-set of G, and a payment rule $p : \Theta^n \to \mathcal{R}_+^n$. For simplicity, we restrict our attention to a deterministic mechanism and assume a mechanism is *almost anonymous*, i.e., obtained results are invariant under permutation of identifiers except for the cases of ties. We use these assumptions to simplify the exposition/notations. Also, we assume a mechanism satisfies consumer sovereignty [5] (a.k.a. *player decisiveness*), i.e., there always exists a type θ_i for bidder i, where bidder i can obtain bundle B. In other words, if bidder i's valuation for B is high enough, then i can obtain B.

In the rest of this paper we mainly use a single-agent model [3], where the reported types of the other bidders except i (denoted as Θ_{-i}) are fixed. In general, an allocation rule is denoted as $X(\theta_i, \Theta_{-i})$, and a payment rule is denoted as $p(\theta_i, \Theta_{-i})$. In the rest of this paper, when using a single-agent model, we abbreviate an allocation rule as $X(\theta_i)$ and a payment rule as $p(\theta_i)$. When discussing some property of the allocation/payment rule in this abbreviated form, we assume the property is true for all Θ_{-i}.

Let us note several desirable properties for combinatorial auction mechanisms.

Definition 2 (individual rationality).
A mechanism $\mathcal{M}(X, p)$ satisfies individual rationality, if $\forall i, \theta_i \in \Theta$, the following condition holds :

$$v(\theta_i, X(\theta_i)) - p(\theta_i) \geq 0. \quad (1)$$

This definition means that no participant suffers any loss in a dominant strategy equilibrium. In this paper, we restrict our attention to individually rational mechanisms.

Definition 3 (strategy-proofness).
A mechanism $\mathcal{M}(X,p)$ is strategy-proof, if $\forall \theta_i, \theta'_i \in \Theta$,

$$v(\theta_i, X(\theta_i)) - p(\theta_i) \geq v(\theta_i, X(\theta'_i)) - p(\theta'_i) \quad (2)$$

holds.

In other words, a mechanism is strategy-proof if reporting true type θ_i is a (weakly) dominant strategy for a bidder.

Next, we introduce a concept called implementability, which is defined on an allocation rule X. In the literature of social choice theory, there have been many works on the implementability of social choice functions [7].

Definition 4 (implementability).
We say an allocation rule X is implementable if there exists a payment rule p such that for every $\theta_i, \theta'_i \in \Theta$, Eq. 2 holds.

If an allocation rule X is implementable, we can find an appropriate payment rule p so that the mechanism $\mathcal{M}(X,p)$ becomes strategy-proof.

Now, we are ready to describe *weak-monotonicity*, which fully characterizes implementable allocation rules [3].

Definition 5 (weak-monotonicity [3]).
An allocation rule X is weakly monotone if for every θ_i, $\theta'_i \in \Theta$, the following condition holds:

$$v(\theta_i, X(\theta_i)) - v(\theta_i, X(\theta'_i)) \geq v(\theta'_i, X(\theta_i)) - v(\theta'_i, X(\theta'_i)). \quad (3)$$

Weak-monotonicity is a very simple condition on allocation rules but it fully characterizes implementable allocation rules [3]; i.e., the following theorem holds in combinatorial auctions assuming the domain is rich.

Theorem 1 (Bikhchandani *et al.* [3]). *An allocation rule X in a combinatorial auction is implementable if and only if X is weakly monotone.*

The following intuitive explanation answers why this theorem holds. The left side of Eq. 3 means the gain of a bidder whose type is θ_i, when he reports his true type θ_i instead of a false type θ'_i. The right side of this equation means the gain of a bidder whose type is θ'_i, when he reports a false type θ_i instead of his true type θ'_i.

If the right side is positive, bidder θ'_i has an incentive to pretend to be θ_i. We can prevent this by charging some payment $p(\theta_i)$ so that truth-telling becomes better for θ'_i; i.e., $v(\theta'_i, X(\theta_i)) - p(\theta_i) - v(\theta'_i, X(\theta'_i)) < 0$ holds.

However, enforcing this payment also reduces the gain of bidder θ_i for truth-telling by $p(\theta_i)$. If Eq. 3 holds, we can always find an appropriate payment rule p so that both $v(\theta_i, X(\theta_i)) - p(\theta_i) - v(\theta_i, X(\theta'_i)) \geq 0$ and $v(\theta'_i, X(\theta_i)) - p(\theta_i) - v(\theta'_i, X(\theta'_i)) < 0$ hold. Similarly, if Eq. 3 does not hold, we can show that finding an appropriate payment rule is impossible.

This theorem indicates that if an allocation rule is weakly monotone, we can always find an appropriate payment rule to implement the allocation rule.

We introduce several notations for false-name-proofness. Let us consider a situation where bidder i uses k false identifiers $id_1, \ldots, id_l, \ldots, id_k$. When an identifier id_l reports his type θ_{id_l}, we represent the allocation rule for id_l as $X_{+I^k_{-l}}(\theta_{id_l}) = X(\theta_{id_l}, \Theta_{-i} \cup I^k_{-l})$, where $I^k_{-l} = \bigcup^k_{j \neq l}\{\theta_{id_j}\}$. Similarly, we represent the payment rule as $p_{+I^k_{-l}}(\theta_{id_l}) = p(\theta_{id_l}, \Theta_{-i} \cup I^k_{-l})$.

Definition 6 (false-name-proofness).
A mechanism $\mathcal{M}(X,p)$ is false-name-proof if for all $k+1$ types of $\theta_i, \theta_{id_1}, \ldots, \theta_{id_l}, \ldots, \theta_{id_k}$,

$$\begin{aligned} &v(\theta_i, X(\theta_i)) - p(\theta_i) \\ &\geq v(\theta_i, \textstyle\bigcup^k_{l=1} X_{+I^k_{-l}}(\theta_{id_l})) - \sum^k_{l=1} p_{+I^k_{-l}}(\theta_{id_l}) \end{aligned} \quad (4)$$

holds.

3. CHARACTERIZATION OF FALSE-NAME-PROOF ALLOCATION RULES

In this section, we introduce a simple condition called *sub-additivity* that fully characterizes false-name-proof allocation rules when coupled with weak-monotonicity. First, we introduce a weaker condition of false-name-proofness, which we call *weak false-name-proofness* (Definition 7) and show that, if a mechanism is strategy-proof and weakly false-name-proof, it is also false-name-proof (Theorem 2). Second, we define a notion called *FN-implementability* (Definition 8), i.e., an allocation rule X is FN-implementable if there exists a payment rule p so that the mechanism $\mathcal{M}(X,p)$ is false-name-proof. Finally, we introduce sub-additivity (Definition 9) and prove that X is FN-implementable if and only if X satisfies weak-monotonicity and sub-additivity (Theorem 3).

Definition 7 (weak false-name-proofness).
A mechanism $\mathcal{M}(X,p)$ is weakly false-name-proof if for all $k+1$ types of $\theta_i, \theta_{id_1}, \ldots, \theta_{id_k}$ such that $X(\theta_i) = \bigcup^k_{l=1} X_{+I^k_{-l}}(\theta_{id_l})$, Eq. 4 holds.

The left side of Eq. 4 indicates the utility of bidder i who truthfully declares his type θ_i using single identifiers. The right side indicates the utility of i who declares his false types $\theta_{id_1}, \ldots, \theta_{id_k}$ using k identifiers $id_1, \ldots, id_k$.

Weak false-name-proofness is weaker than (strong) false-name-proofness, since we assume the obtained goods do not change by using multiple identifiers. However, the next theorem shows that if a mechanism is strategy-proof and weakly false-name-proof, then it is (strongly) false-name-proof.

Theorem 2. *A mechanism is false-name-proof if and only if it is strategy-proof and weakly false-name-proof.*

Proof. Clearly, the definition of false-name-proofness subsumes that of weak false-name-proofness. If we assume a bidder can declare a *null* type, whose valuation of every bundle is zero, and the mechanism never allocates a good to a null type, the definition of false-name-proofness also subsumes strategy-proofness.

We are going to prove that if a mechanism is strategy-proof and weakly false-name-proof, then it is also (strongly) false-name-proof. Assume that a mechanism is not (strongly) false-name-proof; i.e., Eq. 4 does not hold, even though the mechanism is weakly false-name-proof and strategy-proof. More specifically, we assume there exists $k+1$ types θ_i, $\theta_{id_1}, \ldots, \theta_{id_k}$, where the following condition holds:

$$v(\theta_i, X(\theta_i)) - p(\theta_i) < v(\theta_i, \bigcup^k_{l=1} X_{+I^k_{-l}}(\theta_{id_l})) - \sum^k_{l=1} p_{+I^k_{-l}}(\theta_{id_l}). \quad (5)$$

From consumer sovereignty, there exists θ'_i such that $X(\theta'_i) = \bigcup^k_{l=1} X_{+I^k_{-l}}(\theta_{id_l})$. From Eq. 2 for θ_i and θ'_i, the utility of bidder i cannot increase when he declares θ'_i instead of θ_i. From Definition 7, as long as the obtained goods do not change by using multiple identifiers, the payment of θ'_i is smaller than the sum of the payments of multiple identifiers, i.e., $p(\theta'_i) \leq \sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l})$ holds. Accordingly, we obtain

$$\begin{aligned} v(\theta_i, X(\theta_i)) - p(\theta_i) &\geq v(\theta_i, X(\theta'_i)) - p(\theta'_i) \\ &\geq v(\theta_i, X(\theta'_i)) \\ &\quad - \textstyle\sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}) \\ &= v(\theta_i, \textstyle\bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})) \\ &\quad - \textstyle\sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}). \end{aligned}$$

This contradicts Eq. 5. □

Theorem 2 shows that if a strategy-proof mechanism is weakly false-name-proof, it is also (strongly) false-name-proof. Therefore, in the rest of this paper, we restrict our attention to weakly false-name-proof mechanisms.

Next, let us introduce a notion called *false-name(FN)-implementability* for an allocation rule. This definition subsumes the definition of implementability.

Definition 8 (FN-implementability). *An allocation rule X is FN-implementable, if there exists a payment rule p such that for every $\theta_i, \theta'_i \in \Theta$, Eq. 2 holds, and this payment rule satisfies Eq. 4 simultaneously for all $k+1$ types $\theta_i, \theta_{id_1}, \ldots, \theta_{id_k}$, where $X(\theta_i) = \bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})$.*

If an allocation rule X is FN-implementable, we can find an appropriate payment rule p so that the mechanism $\mathcal{M}(X, p)$ becomes weakly false-name-proof. Since Definition 8 assumes that X is implementable, the mechanism $\mathcal{M}(X, p)$ is (strongly) false-name-proof.

Now, we are ready to introduce *sub-additivity*, which fully characterizes FN-implementability when coupled with weak-monotonicity.

Definition 9 (sub-additivity). *An allocation rule X satisfies sub-additivity, if for all $k+1$ types $\theta_i, \theta_{id_1}, \ldots, \theta_{id_k}$ such that $X(\theta_i) = \bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})$,*

$$\begin{array}{lll} \forall \theta'_i & \text{where} & v(\theta'_i, X(\theta'_i)) = 0, \\ \forall \theta'_{id_l} & \text{where} & \left\{ \begin{array}{l} X_{+I^k_{-l}}(\theta'_{id_l}) \supseteq X_{+I^k_{-l}}(\theta_{id_l}), \\ v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta'_{id_l})) \\ \quad = v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})), \end{array} \right. \\ & & (l = 1, 2, \ldots, k) \end{array}$$

$$v(\theta'_i, X(\theta_i)) \leq \sum_{l=1}^{k} v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})) \qquad (6)$$

holds.

An intuitive explanation why sub-additivity holds when an allocation rule X is FN-implementable is as follows. For simplicity, let us assume $k = 2$, i.e., only two false identifiers id_1, id_2 are used. If X is FN-implementable, there exists a payment rule that satisfies $p(\theta_i) \leq p_{+\{\theta_{id_2}\}}(\theta_{id_1}) + p_{+\{\theta_{id_1}\}}(\theta_{id_2})$ (as illustrated in the middle of Figure 1).

The left side of Eq. 6 indicates the valuation of a bidder who declares θ'_i, where his valuation on $X(\theta'_i)$ is zero. Thus, it must be smaller than $p(\theta_i)$; otherwise this bidder has an incentive to pretend that his type is θ_i and to obtain $X(\theta_i)$. This fact is illustrated in the top of Figure 1.

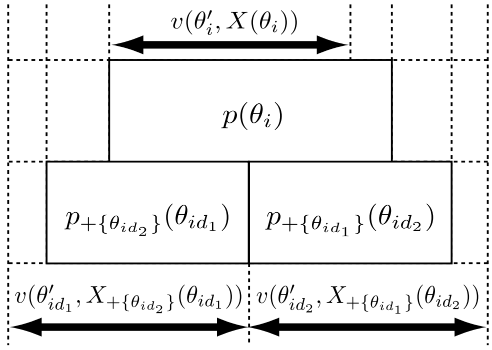

Figure 1: Sub-additivity

Furthermore, if there are only two false identifiers id_1 and id_2, the right side of Eq. 6 becomes $v(\theta'_{id_1}, X_{+\{\theta_{id_2}\}}(\theta_{id_1})) + v(\theta'_{id_2}, X_{+\{\theta_{id_1}\}}(\theta_{id_2}))$, where θ'_{id_1} is a type that can obtain $X_{+\{\theta_{id_2}\}}(\theta_{id_1})$ or any superset. Similarly, θ'_{id_2} can obtain $X_{+\{\theta_{id_1}\}}(\theta_{id_2})$ (or any superset). Then $v(\theta'_{id_1}, X_{+\{id_2\}}(\theta_{id_1}))$ must be greater than $p_{+\{\theta_{id_2}\}}(\theta_{id_1})$; otherwise a bidder with type θ_{id_1} has an incentive to pretend that his type is θ'_{id_1} and to reduce his payment. Similarly, $v(\theta'_{id_2}, X_{+\{\theta_{id_1}\}}(\theta_{id_2}))$ must be greater than $p_{+\{id_1\}}(\theta_{id_2})$. This fact is illustrated in the bottom of Figure 1.

From these facts, the sub-additivity condition must hold. We will show a more rigorous proof in Lemma 1.

Furthermore, as long as the sub-additivity condition and weak-monotonicity hold, we can choose an appropriate payment rule p so that $p(\theta_i) \leq p_{+\{\theta_{id_2}\}}(\theta_{id_1}) + p_{+\{\theta_{id_1}\}}(\theta_{id_2})$ holds. We will show a more detailed explanation in Lemma 2.

Theorem 3. *An allocation rule X is FN-implementable if and only if X satisfies weak-monotonicity and sub-additivity.*

This theorem shows that if an allocation rule satisfies weak-monotonicity and sub-additivity, we can always find a payment rule so that the obtained mechanism is false-name-proof. If it does not satisfy weak-monotonicity or sub-additivity, it is impossible to find an appropriate payment rule. The following lemmas prove this theorem.

Lemma 1. *If an allocation rule X is FN-implementable, X satisfies weak-monotonicity and sub-additivity.*

Proof. Theorem 1 already proves that if an allocation rule X is implementable, it satisfies weak-monotonicity. If X is FN-implementable, it is automatically implementable. Thus, X should satisfy weak-monotonicity.

To prove this lemma, it suffices to show that if X is FN-implementable, it satisfies sub-additivity. We are going to derive a contradiction by assuming Eq. 6 does not hold, although X is FN-implementable and satisfies weak-monotonicity. More specifically, we assume when $X(\theta_i) = \bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})$ holds, there exists types $\theta'_i, \theta'_{id_1}, \ldots, \theta'_{id_k}$, which satisfy the following condition:

$$\begin{array}{lll} \exists \theta'_i & \text{where} & v(\theta'_i, X(\theta'_i)) = 0, \\ \exists \theta'_{id_l} & \text{where} & \left\{ \begin{array}{l} X_{+I^k_{-l}}(\theta'_{id_l}) \supseteq X_{+I^k_{-l}}(\theta_{id_l}), \\ v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta'_{id_l})) \\ \quad = v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})), \end{array} \right. \\ & & (l = 1, 2, \ldots, k) \end{array}$$

$$v(\theta'_i, X(\theta_i)) > \sum_{l=1}^{k} v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})). \qquad (7)$$

Since X is FN-implementable, there exists a payment rule p such that $\mathcal{M}(X,p)$ is strategy/false-name-proof. Since we assume $v(\theta'_i, X(\theta'_i)) = 0$, from individual rationality, $p(\theta'_i) = 0$ holds. From strategy-proofness, $v(\theta'_i, X(\theta'_i)) - p(\theta'_i) = 0 \geq v(\theta'_i, X(\theta_i)) - p(\theta_i)$ holds. Thus, we obtain

$$v(\theta'_i, X(\theta_i)) \leq p(\theta_i). \qquad (8)$$

Also, from the fact that $\mathcal{M}(X,p)$ is false-name-proof, Eq. 4 derives

$$p(\theta_i) \leq \sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}). \qquad (9)$$

Next, to derive

$$\sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}) = \sum_{l=1}^{k} p_{+I^k_{-l}}(\theta'_{id_l}),$$

we are going to show that $p_{+I^k_{-l}}(\theta_{id_l}) = p_{+I^k_{-l}}(\theta'_{id_l})$. Since we assume $v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta'_{id_l})) = v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l}))$, from Eq. 2, we obtain $p_{+I^k_{-l}}(\theta'_{id_l}) \leq p_{+I^k_{-l}}(\theta_{id_l})$. Also, since we assume $X_{+I^k_{-l}}(\theta'_{id_l}) \supseteq X_{+I^k_{-l}}(\theta_{id_l})$ and free disposal, we obtain $v(\theta_{id_l}, X_{+I^k_{-l}}(\theta'_{id_l})) \geq v(\theta_{id_l}, X_{+I^k_{-l}}(\theta_{id_l}))$. Therefore, from Eq. 2, we obtain $p_{+I^k_{-l}}(\theta_{id_l}) \leq p_{+I^k_{-l}}(\theta'_{id_l})$. Thus, the payment of θ'_{id_l} equals that of θ_{id_l}.

Finally, from Eqs.1 and 7, $p_{+I^k_{-l}}(\theta'_{id_l}) \leq v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta'_{id_l})) = v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l}))$ holds for θ'_{id_l} and

$$\sum_{l=1}^{k} p_{+I^k_{-l}}(\theta'_{id_l}) \leq \sum_{l=1}^{k} v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})) \qquad (10)$$

holds. As a result, from Eqs. 8, 9, and 10, we obtain

$$v(\theta'_i, X(\theta_i)) \leq \sum_{l=1}^{k} v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})).$$

This contradicts Eq. 7. □

Lemma 2. *If an allocation rule X satisfies weak-monotonicity and sub-additivity, then X is FN-implementable.*

Proof. Theorem 1 already proves that, if an allocation rule X satisfies weak-monotonicity, it is implementable. Thus, if X satisfies weak-monotonicity, there exists a payment rule p that satisfies Eq. 2. To prove this lemma, we show that if X satisfies sub-additivity, we can choose a payment rule p so that p also satisfies Eq. 4.

We are going to derive a contradiction assuming that an allocation rule X, which satisfies weak-monotonicity and sub-additivity, is not FN-implementable. More specifically, we assume that for any payment rule p that implements X, there exists a bidder with type θ_i, who can increase his profit by using false identifiers $\theta_{id_1}, \ldots, \theta_{id_k}$.

$$\begin{aligned}
&\forall p,\\
&\exists(\theta_i, \theta_{id_1}, \ldots, \theta_{id_k}) \text{ s.t. } X(\theta_i) = \textstyle\bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l}),\\
&v(\theta_i, X(\theta_i)) - p(\theta_i)\\
&\quad < v(\theta_i, \textstyle\bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})) - \textstyle\sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}).
\end{aligned}$$

Since the obtained goods do not change by using multiple identifiers, $p(\theta_i) > \sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l})$ holds. Let us choose γ (> 0) such that

$$p(\theta_i) - \gamma = \sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}) \qquad (11)$$

holds.

Then, let us choose a small enough value ϵ such that $0 < \epsilon < \frac{\gamma}{k+1}$ holds. Also, let us define type θ''_i as follows:

$$v(\theta''_i, Y) = \begin{cases} p(\theta_i) - \epsilon & \text{if } Y \supseteq X(\theta_i), \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, for each $l = 1, \ldots, k$, let θ''_{id_l} be the following type:

$$v(\theta''_{id_l}, Y) = \begin{cases} p_{+I^k_{-l}}(\theta_{id_l}) + \epsilon & \text{if } Y \supseteq X_{+I^k_{-l}}(\theta_{id_l}), \\ 0 & \text{otherwise.} \end{cases}$$

From Lemmas 3 and 4 in Appendix, we can show that those $k+1$ types satisfy the preconditions of sub-additivity. As a result, the following condition holds: for θ''_i and θ''_{id_l},

$$v(\theta''_i, X(\theta_i)) \leq \textstyle\sum_{l=1}^{k} v(\theta''_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})) \ .$$

By substituting the definitions of $v(\theta''_i, \cdot)$ and $v(\theta''_{id_l}, \cdot)$ into this equation, we obtain

$$p(\theta_i) \leq \sum_{l=1}^{k} p_{+I^k_{-l}}(\theta_{id_l}) + (k+1)\epsilon. \qquad (12)$$

Here, by substituting Eq. 11 into Eq. 12, we obtain $\gamma \leq (k+1)\,\epsilon$. Thus, this contradicts the assumption of $\epsilon < \frac{\gamma}{k+1}$. □

4. DISCUSSIONS

The proposed sub-additivity condition enables us to verify whether a mechanism is false-name-proof. In this section, we demonstrate whether the sub-additivity condition is satisfied in several allocation rules.

Claim 1. *A Pareto efficient allocation rule does not satisfy sub-additivity.*

Assume there are two bidders 1 and 2 and two goods a and b for sale in a combinatorial auction. Assume that bidder 2's type is defined as follows:

bidder 2 : $(0, 0, 10)$

Note that $(0, 0, 10)$ represents the valuation of bidder 2 over a, b, and $\{a, b\}$, respectively.

Let us examine an allocation rule for bidder 1. For an allocation rule that achieves Pareto efficiency, bidder 1 obtains $\{a, b\}$ if he has a greater value than 10 on $\{a, b\}$. Thus, if he has a type θ' such that $v(\theta', \{a, b\}) = 10 - \epsilon$ and $v(\theta', \{a\}) = v(\theta', \{b\}) = 0$, he obtains no good, i.e., $X(\theta') = \emptyset$.

On the other hand, let us consider the situation where bidder 1 uses two identifiers: bidders 1′ and 3. The declared types of bidders 1′ and 3 are as follows:

bidder 1′ : $(8, 0, 8)$
bidder 2 : $(0, 0, 10)$
bidder 3 : $(0, 7, 7)$

Since X is Pareto efficient, a is allocated to bidder $1'$ and b is allocated to bidder 3. Thus, for bidder 1, the obtained goods do not change by using two identifiers. Let bidder $1'$ have θ'_1 such that $v(\theta'_1, \{a\}) = 3 + \epsilon$. He is allocated a. Similarly, let bidder 3 have θ_3 such that $v(\theta_3, \{b\}) = 2 + \epsilon$. He is allocated b. Thus, by a Pareto efficient allocation rule, we obtain

$$\begin{aligned} v(\theta', \{a, b\}) = 10 - \epsilon \quad &> \quad v(\theta'_1, \{a\}) + v(\theta_3, \{b\}) \\ &= \quad (3 + \epsilon) + (2 + \epsilon). \end{aligned}$$

Thus, it does not satisfy sub-additivity.

The fact that there exists no false-name-proof, Pareto efficient mechanism is already proved in [20]. However, our proof is much simpler, since we can ignore a payment rule and concentrate on the allocation rule.

Claim 2. *The allocation rule in the Minimal Bundle mechanism [17] satisfies sub-additivity.*

To describe the Minimal-Bundle (MB) mechanism, we first need to define a concept called *minimal bundle).*

Definition 10 (minimal bundle). *Bundle B is called minimal for bidder i if $\forall B' \subset B, B' \neq B, v(\theta_i, B') < v(\theta_i, B)$ holds.*

Notice that a bidder can have multiple minimal bundles. In the MB mechanism, bidder i can obtain bundle B if B is minimal for i and its valuation on B is larger than all the minimal bundles of the other bidders that conflict with B.

The allocation rule can be described as follows:

$$X(\theta_i) = \begin{cases} Y & \text{if there exists } Y s.t.\ Y \text{ is a minimal bundle} \\ & \text{for } i \text{ and } \forall \theta_j \in \Theta_{-i}, \forall Y', \text{ where } Y \cap Y' \neq \emptyset \\ & \text{and } Y' \text{ is a minimal bundle for } j, \\ & v(\theta_i, Y) \geq v(\theta_j, Y') \text{ holds.} \\ \emptyset, & \text{otherwise.} \end{cases}$$

Here, Θ_{-i} represents the set of types of other bidders. More precisely, if multiple minimal bundles of i satisfy this condition, we choose the best bundle that maximizes i's utility. For simplicity, we omit the procedure for tie-breaking.

Let us consider a situation where $X(\theta_i) = \bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})$ holds, and denote $Y = X(\theta_i)$. Let us choose $(\theta_{j^*}, Y'_{max}) = \arg\max_{\theta_j, Y'} v(\theta_j, Y')$, where $Y \cap Y' \neq \emptyset, \forall \theta_j \in \Theta_{-i}$ and Y' is a minimal bundle for j. By choosing θ'_i such that $v(\theta'_i, X(\theta'_i)) = 0$,

$$v(\theta'_i, X(\theta_i)) < v(\theta_{j^*}, Y'_{max})$$

holds.

Since we assume $X(\theta_i) = \bigcup_{l=1}^{k} X_{+I^k_{-l}}(\theta_{id_l})$ holds, for at least one identifier id_h, $X_{+I^k_{-h}}(\theta_{id_h})$ and Y'_{max} share some common element. Furthermore, we can choose θ'_{id_h} such that $X_{+I^k_{-h}}(\theta'_{id_h}) \supseteq X_{+I^k_{-h}}(\theta_{id_h})$, $v(\theta'_{id_h}, X_{+I^k_{-h}}(\theta_{id_h})) \geq v(\theta_{j^*}, Y'_{max})$ holds.

As a result, we obtain

$$\begin{aligned} v(\theta'_i, X(\theta_i)) \quad &< \quad v(\theta_{j^*}, Y'_{max}) \\ &\leq \quad v(\theta'_{id_h}, X_{+I^k_{-h}}(\theta_{id_h})) \\ &\leq \quad v(\theta'_{id_h}, X_{+I^k_{-h}}(\theta_{id_h})) \\ &\quad\quad + \textstyle\sum_{l \neq h} v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})) \\ &= \quad \textstyle\sum_{l=1}^{k} v(\theta'_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})). \end{aligned}$$

Thus, sub-additivity condition is satisfied.

Claim 3. *The allocation rule in the Leveled Division Set mechanism [19] satisfies sub-additivity.*

The Leveled Division Set mechanism is very complicated. For simplicity, we describe the allocation rule when there are only two goods a and b, and the reserve price for each is r.

$$X(\theta_i) = \begin{cases} \{a, b\} & \text{if } v(\theta_i, \{a, b\}) \geq 2 \times r \text{ and} \\ & \forall \theta_j \in \Theta_{-i}, v(\theta_i, \{a, b\}) \geq v(\theta_j, \{a, b\}), \\ \{a\} & \text{if } v(\theta_i, \{a\}) \geq r \text{ and} \\ & \forall \theta_j \in \Theta_{-i}, v(\theta_j, \{a, b\}) < 2 \times r, \text{ and} \\ & v(\theta_i, \{a\}) \geq v(\theta_j, \{a\}), \\ \{b\} & \text{if } v(\theta_i, \{b\}) \geq r \text{ and} \\ & \forall \theta_j \in \Theta_{-i}, v(\theta_j, \{a, b\}) < 2 \times r, \text{ and} \\ & v(\theta_i, \{b\}) \geq v(\theta_j, \{b\}), \\ \emptyset, & \text{otherwise.} \end{cases}$$

It is clear that X satisfies sub-additivity when $X(\theta_i)$ is a or b. Thus, we show that X satisfies sub-additivity when $X(\theta_i) = \{a, b\}$. Let us assume that $X_{+\{\theta_{id_2}\}}(\theta_{id_1}) = \{a\}$, and $X_{+\{\theta_{id_1}\}}(\theta_{id_2}) = \{b\}$. Therefore, there exists no θ_j such that $v(\theta_j, \{a, b\}) \geq 2 \times r$ for $\theta_j \in \Theta_{-i}$. Therefore, for θ'_i such that $v(\theta'_i, X(\theta'_i)) = 0$, $v(\theta'_i, \{a, b\})) < 2 \times r$ always holds.

On the other hand, it is clear that for types θ'_{id_1} and θ'_{id_2} such that $X_{+\{\theta_{id_2}\}}(\theta'_{id_1}) = \{a\}$ and $X_{+\{\theta_{id_1}\}}(\theta'_{id_2}) = \{b\}$, $v(\theta'_{id_1}, \{a\}) \geq r$ and $v(\theta'_{id_2}, \{b\}) \geq r$ must hold.

Accordingly, we obtain

$$\begin{aligned} v(\theta'_i, X(\theta_i)) \quad &< \quad 2 \times r \\ &\leq \quad v(\theta'_{id_1}, X_{+\{\theta_{id_2}\}}(\theta_{id_1})) \\ &\quad\quad + v(\theta'_{id_2}, X_{+\{\theta_{id_1}\}}(\theta_{id_2})), \end{aligned}$$

and sub-additivity is satisfied.

Claim 4. *The allocation rule in the GM-SMA [18] does not satisfy sub-additivity.*

The allocation of GM-SMA is based on a Pareto efficient allocation. However, the mechanism adjusts the payment of bidder i who obtains bundle $B \subseteq G$ based on *sub-modular approximation* defined as follows:

$$p(\theta_i, B) = U^*(\Theta_{-i}, G) - V^*(\Theta_{-i}, G \setminus B).$$

Here, $V^*(\Theta_{-i}, G \setminus B)$ is a social surplus when optimally allocating goods $G \setminus B$ for bidders Θ_{-i}. $U^*(\Theta_{-i}, G)$ is defined a similar way but we approximate bidders' valuations so that they satisfy *sum-modularity* defined as follows.

Definition 11 (sub-modularity). *For all $N' \subseteq N$, $B', B'' \subseteq G$, the following condition holds:*

$$\begin{aligned} &U^*(\Theta_{N'}, B') + U^*(\Theta_{N'}, B'') \\ &\geq U^*(\Theta_{N'}, B' \cup B'') + U^*(\Theta_{N'}, B' \cap B''). \end{aligned}$$

The payment of GM-SMA is larger than (or at least equal to) the VCG payment. If payment $p(\theta_i, B)$ becomes larger than valuation $v(\theta_i, B)$, bundle B is not allocated to bidder i.

Assume there are two goods a and b. Let us consider the allocation rule for bidder 1, given the type of bidder 2 described as follows:

$$\text{bidder } 2 : (0, 0, 10)$$

This valuation does not satisfy sub-modularity; i.e., for bidder 2, the valuations for a only and b only are 0, while having a and b together is 10. Let us assume we approximate this valuation as follows[1]:

$$\text{bidder } 2 : (5, 5, 10)$$

If bidder 1 is interested in a only, he cannot obtain it when his (declared) valuation for a is less than 10. In this case, let us consider a type θ_1 such that $v(\theta_1, \{a\}) = 10 - \epsilon$. Then, $X(\theta_1) = \emptyset$ and $v(\theta_1, X(\theta_1)) = 0$ hold.

Now, let us consider another situation where bidder 1 uses two identifiers $1'$ and 3, and declares the following types:

$$\begin{array}{lcl} \text{bidder } 1' & : & (10-\epsilon, 0, 10-\epsilon) \\ \text{bidder } 2 & : & (0, 0, 10) \\ \text{bidder } 3 & : & (0, 5-\epsilon, 5-\epsilon) \end{array}$$

Let us choose a type $\theta_{1'}$ such that $v(\theta_{1'}, \{a\}) = 5 + 2\epsilon$. When bidders 2 and 3 exist, a is allocated to $\theta_{1'}$. Also, let us choose a type θ_3 such that $v(\theta_3, \{b\}) = 0$. When bidders 1' and 2 exist, $\emptyset$ is allocated to θ_3. Therefore, we obtain

$$\begin{array}{rcl} v(\theta_1, \{a\}) = 10 - \epsilon & > & v(\theta_{1'}, \{a\}) + v(\theta_3, \emptyset) \\ & = & 5 + 2\epsilon + 0. \end{array}$$

Thus, this allocation rule does not satisfy sub-additivity.

This fact means that there exists no payment rule p that the mechanism $\mathcal{M}(X, p)$ is false-name-proof. In the initial example, if the valuation of bidder 1 for a is 9, the allocation rule allocates $\emptyset$ for bidder 1 and his payment is 0. On the other hand, in the second example where bidder 1 uses two identifiers, bidder $1'$ obtains a and bidder 3 obtains $\emptyset$ (since the payment with sub-modular approximation is 5, which is larger than bidder 3's valuation $5 - \epsilon$). Thus, the payments of $1'$ and 3 are 5 and 0, respectively. As a result, bidder 1 can increase his utility using false-names. Thus, GM-SMA is not false-name-proof.

Claim 5. *The allocation rule in the Matsuo mechanism [8] does not satisfy sub-additivity.*

The Matsuo mechanism first finds a Pareto efficient allocation. Then, it identifies a set of bidders (called shill group), who can potentially serve as false-names of a single bidder. More specifically, when excluding bidder i, if another bidder j no longer belongs to the Pareto efficient allocation, then the mechanism regards i and j as a shill group. Next, it assumes a set of goods that are allocated to the members of the shill group as a single good, and re-calculates a Pareto efficient allocation. If goods a and b are assumed as a single good, then to obtain good a, a bidder needs to defeat other bidders who want b and $\{a, b\}$, as well as a.

Assume there are two goods a and b. Let us consider the allocation rule for bidder 1, given the types of bidders 2 and 3 described as follows:

$$\begin{array}{ll} \text{bidder } 2 : & (0, 0, 10) \\ \text{bidder } 3 : & (0, 9, 9) \end{array}$$

If bidder 1 is interested in a only, he cannot obtain it when his (declared) valuation for a is less than 10. For example, if his valuation is 9, he is originally included in a Pareto efficient allocation. However, bidders 1 and 3 are considered as a shill group, since by excluding bidder 3, bidder 1 is no longer in the Pareto efficient allocation. Since bidder 1 needs to defeat bidder 2 to obtain a, his valuation for a must be larger than 10. In this case, let us consider a type θ' such that $v(\theta', \{a\}) = 10 - \epsilon$. Then, $X(\theta') = \emptyset$ and $v(\theta', X(\theta')) = 0$ hold.

Now, let us consider another situation where bidder 1 uses three identifiers, $1'$, 4, and 5:

$$\begin{array}{ll} \text{bidder } 1' : & (9, 0, 9) \\ \text{bidder } 2 : & (0, 0, 10) \\ \text{bidder } 3 : & (0, 9, 9) \\ \text{bidder } 4 : & (2, 0, 2) \\ \text{bidder } 5 : & (0, 8, 8) \end{array}$$

In this case, bidders $1'$ and 3 are originally included in the Pareto efficient allocation. Even if we exclude bidder 3 (or bidder 1), bidder 1 (or bidder 3) remains in the Pareto efficient allocation. Thus, no shill group is identified.

Let us choose a type θ'_1 such that $v(\theta'_1, \{a\}) = 2+\epsilon$. When bidders 2–5 exist, a is allocated to θ'_1. Also, let us choose a type θ'_4 such that $v(\theta'_4, \{a\}) = 0$. When bidders 1–3 and 5 exist, $\emptyset$ is allocated to θ'_4. Furthermore, let us choose a type θ'_5 such that $v(\theta'_5, \{b\}) = 0$. When bidders 1–3 and 4 exist, $\emptyset$ is allocated to θ'_5.

Therefore, we obtain

$$\begin{array}{rcl} v(\theta', \{a\}) & = & 10 - \epsilon \\ & > & v(\theta'_1, \{a\}) + v(\theta'_4, \emptyset) + v(\theta'_5, \emptyset) \\ & = & 2 + \epsilon + 0 + 0. \end{array}$$

Thus, this allocation rule does not satisfy sub-additivity. This fact means that there exists no payment rule that satisfies FN-implementability for this allocation rule. The Matsuo mechanism uses VCG payment. In the initial example, if the valuation of bidder 1 for a is 9, the allocation rule allocates $\emptyset$ for bidder 1 and his payment is 0. On the other hand, in the second example where bidder 1 uses three identifiers, bidder $1'$ obtains a and both bidders 4 and 5 obtain $\emptyset$. The payments of $1'$, 4, and 5 are 2, 0, and 0, respectively. As a result, bidder 1 can increase his utility using false-names.

It is intuitively natural to believe that GM-SMA and Matsuo mechanism are false-name-proof, since false-name manipulation is not profitable in these mechanisms as long as the goods obtained by each false identifier are non-empty (e.g., a bidder who wants $\{a, b\}$ obtains a with one identifier and b with another identifier). However, in the above examples, a bidder can decrease his payment by submitting false-name-bids that do not receive any good.

Detecting such a subtle failure of a mechanism is quite difficult if we rely on standard proof techniques for false-name-proofness. By utilizing the sub-additivity condition introduced in this paper, we can easily verify whether a mechanism is false-name-proof.

5. CONCLUSIONS

We identified a simple condition called *sub-additivity*, which characterizes false-name-proof allocation rules of combinatorial auctions. We proved that an allocation rule is FN-implementable, i.e., we can construct a false-name-proof mechanism based on the allocation rule, if and only if it satisfies weak-monotonicity and sub-additivity.

As far as the authors are aware, this is the first attempt to characterize false-name-proof allocation rules. Our proposed characterization is useful for developing new mechanisms or for verifying existing mechanisms. To demonstrate the power of our characterization, we verified existing false-

[1]We can construct a similar example for different approximations.

name-proof mechanisms and found that two mechanisms, which were believed to be false-name-proof, are actually not.

In future works, we hope to examine several theoretical properties of false-name-proof allocation rules, e.g., the upper-bound of possible social surplus for false-name-proof mechanisms, by utilizing our characterization. Furthermore, we would like to extend our characterization to broader social choice functions such as voting and collective decision-making. Finally, we are planning to examine the characterization of allocation rules that satisfy robustness against other manipulations, such as group-strategy-proofness and collusion-proofness [11, 9].

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APPENDIX

Lemma 3. *Assume a payment rule p implements an allocation rule X. Also, let us assume a type θ''_i satisfies the following condition:*

$$v(\theta''_i, Y) = \begin{cases} p(\theta_i) - \epsilon & \text{if } Y \supseteq X(\theta_i), \\ 0 & \text{otherwise.} \end{cases}$$

Then $v(\theta''_i, X(\theta''_i)) = 0$ holds.

Proof. To prove this lemma, we show that $X(\theta''_i) \not\supseteq X(\theta_i)$ for θ''_i holds. We derive a contradiction by assuming $X(\theta''_i) \supseteq X(\theta_i)$ holds. Since we assume the mechanism is individually rational (Definition 2), the following condition must hold:

$$\begin{aligned} p(\theta''_i) &\leq v(\theta''_i, X(\theta''_i)) \\ &= p(\theta_i) - \epsilon. \end{aligned}$$

From free disposal and $X(\theta''_i) \supseteq X(\theta_i)$, $v(\theta_i, X(\theta''_i)) \geq v(\theta_i, X(\theta_i))$ holds. By substituting this into Eq. 2, we obtain

$$p(\theta_i) \leq p(\theta''_i).$$

Thus, $p(\theta_i) \leq p(\theta''_i) \leq p(\theta_i) - \epsilon$ holds. This contradicts the assumption of $\epsilon > 0$. □

Lemma 4. *Assume a payment rule p implements an allocation rule X. Also, let us assume types θ''_{id_l} ($l = (1, \ldots, k)$) satisfy the following conditions:*

$$v(\theta''_{id_l}, Y) = \begin{cases} p_{+I^k_{-l}}(\theta_{id_l}) + \epsilon & \text{if } Y \supseteq X_{+I^k_{-l}}(\theta_{id_l}), \\ 0 & \text{otherwise.} \end{cases}$$

Then $X_{+I^k_{-l}}(\theta''_{id_l}) \supseteq X_{+I^k_{-l}}(\theta_{id_l})$ holds. Thus, $v(\theta''_{id_l}, X_{+I^k_{-l}}(\theta''_{id_l})) = v(\theta''_{id_l}, X_{+I^k_{-l}}(\theta_{id_l}))$ also holds.

Proof. We derive a contradiction by assuming $X_{+I^k_{-l}}(\theta''_{id_l}) \not\supseteq X_{+I^k_{-l}}(\theta_{id_l})$. By the above assumption, $v(\theta''_{id_l}, X_{+I^k_{-l}}(\theta''_{id_l})) = 0$. Since we assume the mechanism is individually rational, $p_{+I^k_{-l}}(\theta''_{id_l}) = 0$ holds. Since p implements X, the following condition holds (otherwise, θ''_{id_l} has an incentive to pretend to be θ_{id_l}): $v(\theta''_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})) - p_{+I^k_{-l}}(\theta_{id_l}) \leq 0$. From the above condition, we derive

$$\begin{aligned} p_{+I^k_{-l}}(\theta_{id_l}) &\geq v(\theta''_{id_l}, X_{+I^k_{-l}}(\theta_{id_l})) \\ &= p_{+I^k_{-l}}(\theta_{id_l}) + \epsilon. \end{aligned}$$

This contradicts the assumption of $\epsilon > 0$. Thus, for θ''_{id_l}, $X_{+I^k_{-l}}(\theta''_{id_l}) \supseteq X_{+I^k_{-l}}(\theta_{id_l})$ always holds. □